UNIT -1

1\. Define an orthogonal Matrix and solve

$$A = \frac{1}{2}\begin{bmatrix} -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 \end{bmatrix}$$

Sol: Given $A = \frac{1}{2}\begin{bmatrix} -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 \end{bmatrix}$

$$\therefore A^T = \frac{1}{2}\begin{bmatrix} -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 \end{bmatrix}$$

$$A \cdot A^T = \frac{1}{4}\begin{bmatrix} -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 \end{bmatrix}\begin{bmatrix} -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = I_4$$

$$\therefore A \cdot A^T = I_4$$

A is orthogonal.

(2) show that every square matrix can be be expressed as a sum of Hermitian

and skew Hermitian matrices in one and only way.

Sol: Let A be any Square matrix.

consider $A = \frac{1}{2}(A+A^\theta) + \frac{1}{2}(A-A^\theta)$

$\Rightarrow A = P+Q$ where $P = \frac{1}{2}(A+A^\theta)$, $Q = \frac{1}{2}(A-A^\theta)$

Now we have to P.T P is hermitian & Q is skew hermitian.

i.e to P.T $P^\theta = P$ | $Q^\theta = -Q$.

consider $P^\theta = \left[\frac{1}{2}(A+A^\theta)\right]^\theta$

$= \frac{1}{2}\left[A+A^\theta\right]^\theta = \frac{1}{2}\left[A^\theta + (A^\theta)^\theta\right]$

$= \frac{1}{2}\left[A^\theta + A\right]$ $(\because (A^\theta)^\theta = A)$

$= P.$

$P^\theta = P$

$Q^\theta = \left[\frac{1}{2}(A-A^\theta)\right]^\theta = \frac{1}{2}(A-A^\theta)^\theta$

$= \frac{1}{2}\left[A^\theta - (A^\theta)^\theta\right] = \frac{1}{2}\left[A^\theta - A\right] = -\frac{1}{2}\left[A - A^\theta\right]$

$= -Q.$

$\therefore P^\theta = P \Rightarrow P$ is Hermitian | $Q^\theta = -Q \Rightarrow Q$ is skew hermitian.

To P.T the representation is unique:-

Let $A = R+S$ be another such representation of A, where R is Hermitian and S is skew-Hermitian. Then to P.T $R=P$ & $S=Q$

Then $A^\theta = (R+S)^\theta = R^\theta + S^\theta = R - S$ ($\because R$ is Hermitian & S is skew Hermitian)

$\therefore R = \frac{1}{2}(A+A^\theta) = P$ & $S = \frac{1}{2}(A-A^\theta) = Q \Rightarrow$ Thus the representation is unique.

(3) S.T $\begin{bmatrix} a+ic & -b+id \\ b+id & a-ic \end{bmatrix}$ is unitary $a^2+b^2+c^2+d^2=1$.

Sol: Given $A = \begin{bmatrix} a+ic & -b+id \\ b+id & a-ic \end{bmatrix}$

$$\bar{A} = \begin{bmatrix} a-ic & -b-id \\ b-id & a+ic \end{bmatrix}.$$

$$\Rightarrow A^{\theta} = (\bar{A})^T = \begin{bmatrix} a-ic & b-id \\ -b-id & a+ic \end{bmatrix}.$$

Then,

$$AA^{\theta} = \begin{bmatrix} a+ic & -b+id \\ b+id & a-ic \end{bmatrix} \begin{bmatrix} a-ic & b-id \\ -b-id & a+ic \end{bmatrix}$$

$$= \begin{bmatrix} a^2+b^2+c^2+d^2 & 0 \\ 0 & a^2+b^2+c^2+d^2 \end{bmatrix}.$$

$\therefore AA^{\theta} = I$ if and only if $a^2+b^2+c^2+d^2 = 1$.

$\therefore$ A is Unitary if and only if $a^2+b^2+c^2+d^2 = 1$.

4) find the value of 'k' if the rank of matrix A is 2.

Where $A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & k & 0 \end{bmatrix}$

sol: The Given matrix is $A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & k & 0 \end{bmatrix}$

$$\Rightarrow \sim \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & -1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & k & 0 \end{bmatrix} \qquad R_1 \longleftrightarrow R_2.$$

$$\Rightarrow \sim \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & -1 \\ 0 & 1 & -3 & -1 \\ 0 & 1 & k-1 & -1 \end{bmatrix} \qquad \begin{matrix} R_3 \to R_3 - 3R_1. \\ R_4 \to R_4 - R_1. \end{matrix}$$

for the rank $A = 2$, we must have three identical rows.

$$\Rightarrow \therefore k-1 = -3 \Rightarrow k = -2.$$

5) a) find the rank of the matrix $A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$ by reducing to echelon form.

b) find the rank of $\begin{bmatrix} 1 & 4 & 3 & -2 & 1 \\ -2 & -3 & -1 & 4 & 3 \\ -1 & 6 & 7 & 2 & 9 \\ -3 & 3 & 6 & 6 & 12 \end{bmatrix}$.

c) $\begin{bmatrix} 1 & 2 & 3 & 4 \\ -2 & -3 & 1 & 2 \\ -3 & -4 & 5 & 8 \\ 1 & 3 & 10 & 14 \end{bmatrix}$ find the Rank.

Sol:- a):

$$\text{let } A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & -1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix} \quad R_1 \leftrightarrow R_2.$$

$$\sim \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & -1 \\ 0 & 1 & -3 & -1 \\ 0 & 1 & -3 & -1 \end{bmatrix} \quad \begin{matrix} R_3 \rightarrow R_3 - 3R_1 \\ R_4 \rightarrow R_4 - R_1. \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{matrix} R_4 \rightarrow R_4 - R_2 \\ R_3 \rightarrow R_3 - R_2. \end{matrix}$$

This is in the Echelon form.

$\therefore$ Rank of the matrix = 2.

Because the non-zero rows is 2.

sol:- b)

let $A = \begin{bmatrix} 1 & 4 & 3 & -2 & 1 \\ -2 & -3 & -1 & 4 & 3 \\ -1 & 6 & 7 & 2 & 9 \\ -3 & 3 & 6 & 6 & 12 \end{bmatrix}$.

$\sim \begin{bmatrix} 1 & 4 & 3 & -2 & 1 \\ 0 & 5 & 5 & 0 & 5 \\ 0 & 10 & 10 & 0 & 10 \\ 0 & 15 & 15 & 0 & 15 \end{bmatrix}$ $R_2 \to R_2 + 2R_1$, $R_3 \to R_3 + R_1$, $R_4 \to R_4 + 3R_1$.

$\sim \begin{bmatrix} 1 & 4 & 3 & -2 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 \end{bmatrix}$ $R_2 \to \frac{R_2}{5}$ $R_3 \to \frac{R_3}{10}$. $R_4 \to \frac{R_4}{15}$.

$\sim \begin{bmatrix} 1 & 4 & 3 & -2 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ $R_3 \to R_3 - R_2$, $R_4 \to R_4 - R_2$.

$\therefore$ Rank $(A) = \rho(A) = 2$.

$\therefore$ The no, of non-zero rows in the matrix $= 2$.

(b)

Sol:

Let $A = \begin{bmatrix} 1 & 4 & 3 & -2 & 1 \\ -2 & -3 & -1 & 4 & 3 \\ -1 & 6 & 7 & 2 & 9 \\ -3 & 3 & 6 & 6 & 12 \end{bmatrix}$

$\sim \begin{bmatrix} 1 & 4 & 3 & -2 & 1 \\ 0 & 5 & 5 & 0 & 5 \\ 0 & 10 & 10 & 0 & 10 \\ 0 & 15 & 15 & 0 & 15 \end{bmatrix}$ $\quad R_2 \to R_2 + 2R_1,\ R_3 \to R_3 + R_1,\ R_4 \to R_4 + 3R_1.$

$\sim \begin{bmatrix} 1 & 4 & 3 & -2 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 \end{bmatrix}$ $\quad R_2 \to \frac{R_2}{5},\ R_3 \to \frac{R_3}{10},\ R_4 \to \frac{R_4}{15}.$

$\sim \begin{bmatrix} 1 & 4 & 3 & -2 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ $\quad R_3 \to R_3 - R_2,\ R_4 \to R_4 - R_2.$

$\therefore$ rank $(A) = \rho(A) = 2$

$\because$ The no. of non-zero rows in the matrix $= 2$.

6. Find the rank of $\begin{bmatrix} 1 & 4 & 3 & -2 & 1 \\ -2 & -3 & -1 & 4 & 3 \\ -1 & 6 & 7 & 2 & 9 \\ -3 & 3 & 6 & 6 & 12 \end{bmatrix}$

Let $A = \begin{bmatrix} 1 & 4 & 3 & -2 & 1 \\ -2 & -3 & -1 & 4 & 3 \\ -1 & 6 & 7 & 2 & 9 \\ -3 & 3 & 6 & 6 & 12 \end{bmatrix}$

Sol:- $R_2 \to R_2 + 2R_1$

$R_3 \to R_3 + R_1$

$R_4 \to R_4 + 3R_1$

$\Rightarrow \begin{bmatrix} 1 & 4 & 3 & -2 & 1 \\ 0 & 5 & 5 & 0 & 5 \\ 0 & 10 & 10 & 0 & 10 \\ 0 & 15 & 15 & 0 & 15 \end{bmatrix}$

$R_3 \to R_3 - 2R_2$

$R_4 \to R_4 - 3R_2$

$\Rightarrow \begin{bmatrix} 1 & 4 & 3 & -2 & 1 \\ 0 & 5 & 5 & 0 & 5 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 & 4 & 3 & -2 & 1 \\ 0 & 5 & 5 & 0 & 5 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

The Number of Non-zero rows $= 2$.

$\therefore \rho(A) = 2$

7. Find the rank of $\begin{bmatrix} 1 & 2 & 3 & 4 \\ -2 & -3 & 1 & 2 \\ -3 & -4 & 5 & 8 \\ 1 & 3 & -10 & 14 \end{bmatrix}$

Let $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ -2 & -3 & 1 & 2 \\ -3 & -4 & 5 & 8 \\ 1 & 3 & -10 & 14 \end{bmatrix}$

Sol:- $R_2 \to R_2 + 2R_1$

$R_3 \to R_3 + 3R_1$

$R_4 \to R_4 - R_1$

$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 7 & 10 \\ 0 & 2 & 14 & 20 \\ 0 & 1 & 7 & 10 \end{bmatrix}$

$R_3 \to R_2 - 2R_2$

$\Rightarrow$ $R_4 \to R_4 - R_2$ $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 7 & 10 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$\Rightarrow$ $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 7 & 10 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

The number of Non-zero rows are 2.

$\therefore \rho(A) = 2$

8. What is Normal form of a matrix and find the rank of given matrix. $A = \begin{bmatrix} 2 & 1 & 3 & 4 \\ 0 & 3 & 4 & 1 \\ 2 & 3 & 7 & 5 \\ 2 & 5 & 11 & 6 \end{bmatrix}$

Sol:- Every $m \times n$ order matrix can be reduced into any one of the following forms. I_r, $[I_r, 0]$, $\begin{bmatrix} I_r \\ 0 \end{bmatrix}$ $\begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$ by using elementary transformation rows and columns, where I_r means Identity matrix of order r.

$R_3 \to R_3 - R_1$

$R_4 \to R_4 - R_1$ $\begin{bmatrix} 2 & 1 & 3 & 4 \\ 0 & 3 & 4 & 1 \\ 0 & 2 & 4 & 1 \\ 0 & 4 & 8 & 2 \end{bmatrix}$

$R_4 \to R_4 - R_2$ $\begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 5 & 3 & 7 \\ 0 & 4 & 9 & 10 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$R_3 \to 5R_3 - 4R_2$ $\begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 5 & 3 & 7 \\ 0 & 0 & 33 & 22 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$R_3/11$ $\begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 5 & 3 & 7 \\ 0 & 0 & 3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$C_2 \to C_2 + C_1$
$C_3 \to C_3 + 2C_1$
$C_4 \to C_4 + 4C_1$ $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 5 & 3 & 7 \\ 0 & 0 & 3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$C_3 \to 5C_3 - 3C_2$
$C_4 \to 5C_4 - 7C_2$ $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 15 & 10 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$R_2/5$
$R_3/5$ $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$C_4 \to 3C_4 - 2C_3$ $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$\begin{bmatrix} I_3 & 0 \\ 0 & 0 \end{bmatrix}$ //

$R_3 \to 3R_3 - 2R_2$
$R_4 \to 3R_4 - 4R_2$
$$\begin{bmatrix} 2 & 1 & 3 & 4 \\ 0 & 3 & 4 & 1 \\ 0 & 0 & 4 & 1 \\ 0 & 0 & 8 & 2 \end{bmatrix}$$

$R_4 \to R_4 - 2R_3$
$$\begin{bmatrix} 2 & 1 & 3 & 4 \\ 0 & 3 & 4 & 1 \\ 0 & 0 & 4 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Number of Non-zero rows are 3.

$\therefore \rho(A) = 3$

9. By reducing the matrix $\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$ into normal form and find it's rank.

Sol:- let $A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$

$R_2 \to R_2 - 2R_1$
$R_3 \to R_3 - 3R_1$
$R_4 \to R_4 - 6R_1$
$$\begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 5 & 3 & 7 \\ 0 & 4 & 9 & 10 \\ 0 & 9 & 12 & 17 \end{bmatrix}$$

$R_3 \to 5R_3 - 4R_2$
$R_4 \to 5R_4 - 9R_2$
$$\begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 5 & 3 & 7 \\ 0 & 4 & 9 & 10 \\ 0 & 5 & 3 & 7 \end{bmatrix}$$

10. $A = \begin{bmatrix} 2 & 1 & 3 & 5 \\ 4 & 2 & 1 & 3 \\ 8 & 4 & 7 & 13 \\ 8 & 4 & -3 & -1 \end{bmatrix}$. reduce to normal form and find rank.

Sol:- let $A = \begin{bmatrix} 2 & 1 & 3 & 5 \\ 4 & 2 & 1 & 3 \\ 8 & 4 & 7 & 13 \\ 8 & 4 & -3 & -1 \end{bmatrix}$

$R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 4R_1$, $R_4 \to R_4 - 4R_1$

$$\sim \begin{bmatrix} 2 & 1 & 3 & 5 \\ 0 & 0 & -5 & -7 \\ 0 & 0 & -5 & -7 \\ 0 & 0 & -15 & -21 \end{bmatrix}$$

$R_4 \to R_4 - 3R_3$, $R_3 \to R_3 - R_2$

$$\sim \begin{bmatrix} 2 & 1 & 3 & 5 \\ 0 & 0 & -5 & -7 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_1 \to 5R_1 + 3R_2$

$$\sim \begin{bmatrix} 10 & 5 & 0 & 4 \\ 0 & 0 & -5 & -7 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\frac{C_1}{10}, \frac{C_2}{5}, \frac{C_3}{-5}$

$$\sim \begin{bmatrix} 1 & 1 & 0 & 4 \\ 0 & 0 & 1 & -7 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_2 - C_1$, $C_4 - 4C_1$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -7 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_4 \to C_4 + 7C_3$ $\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$C_2 \leftrightarrow C_3$ $\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$\begin{bmatrix} I_2 & 0 \\ 0 & 0 \end{bmatrix}$

This is of the form $\begin{bmatrix} I_2 & 0 \\ 0 & 0 \end{bmatrix}$

$\therefore$ Rank of $A = 2$

17) find the Inverse of the Matrix A using elementary operation (Gauss-Jordan Method).

i) $A = \begin{bmatrix} -1 & -3 & 3 & -1 \\ 1 & 1 & -1 & 0 \\ 2 & -5 & 2 & -3 \\ -1 & 1 & 0 & 1 \end{bmatrix}$

i.e $\begin{bmatrix} -1 & -3 & 3 & -1 \\ 1 & 1 & -1 & 0 \\ 2 & -5 & 2 & -3 \\ -1 & 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} A$

⇒ We will Apply row operations on matrix A in LHS.

⇒ Applying $R_2 \to R_2 + R_1$, $R_3 \to R_3 + 2R_1$ and $R_4 \to R_4 - R_1$, we get

$$\begin{bmatrix} -1 & -3 & 3 & -1 \\ 0 & -2 & 2 & -1 \\ 0 & -11 & 8 & -5 \\ 0 & 4 & -3 & 2 \end{bmatrix} = \begin{bmatrix} -1 & -3 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ -7 & -11 & 2 & 0 \\ 1 & 2 & 0 & 1 \end{bmatrix} A$$

⇒ Applying $R_2 \to R_2 - 2R_4$ and $R_3 \to R_3 + 6R_4$, we get

$$\begin{bmatrix} -2 & 0 & 0 & 1 \\ 0 & -2 & 0 & -1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} -1 & -3 & 0 & 0 \\ -1 & -3 & 0 & -2 \\ -1 & 1 & 2 & 6 \\ 1 & 2 & 0 & 1 \end{bmatrix}$$

⇒ Applying $R_1 \to R_1 - R_3$ and $R_2 \to R_2 + R_3$, we get

$$\begin{bmatrix} -2 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -4 & -2 & -6 \\ -2 & -2 & 2 & 4 \\ -1 & 1 & 2 & 6 \\ 1 & 2 & 0 & 1 \end{bmatrix} A$$

⇒ Applying $R_1/_{-2}$, $R_2/_{-2}$ and $R_3 \leftrightarrow R_4$, We get.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 2 & 1 & 3 \\ 1 & 1 & -1 & -2 \\ 1 & 2 & 0 & 1 \\ -1 & 1 & 2 & 6 \end{bmatrix} A.$$

This is of the form $I_4 = BA$.

$$A^{-1} = B = \begin{bmatrix} 0 & 2 & 1 & 3 \\ 1 & 1 & -1 & -2 \\ 1 & 2 & 0 & 1 \\ -1 & 1 & 2 & 6 \end{bmatrix}.$$

ii) $B = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$

Solution: We write $A = I_3 A$.

We perform elementary row operation on LHS to reduce it to I_3

$\Rightarrow$ $I_3 = BA$. Then

$\Rightarrow$ $B = A^{-1}$.

$$\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \to R_2 - R_1$, $R_3 \to R_3 + 2R_1$ we get.

$$\begin{bmatrix} 1 & 1 & 3 \\ 0 & 2 & -6 \\ 0 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} A$$

Applying $\frac{R_2}{2}$, and $\frac{R_3}{2}$, we get

$$\begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & -3 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1/2 & 1/2 & 0 \\ 1 & 0 & 1/2 \end{bmatrix} A$$

Applying $R_1 \to R_1 - R_2$ and $R_3 \to R_3 + R_2$, we get

$$\begin{bmatrix} 1 & 0 & 6 \\ 0 & 1 & -3 \\ 0 & 0 & -2 \end{bmatrix} = \begin{bmatrix} 3/2 & -1/2 & 0 \\ -1/2 & 1/2 & 0 \\ 1/2 & 1/2 & 1/2 \end{bmatrix} A$$

$$\begin{bmatrix} 1 & 0 & 6 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3/2 & -1/2 & 0 \\ -1/2 & 1/2 & 0 \\ -1/4 & -1/4 & -1/4 \end{bmatrix} A \quad \left(\text{Applying } R_3 \to \frac{R_3}{-2}\right)$$

Applying $R_1 \to R_1 - 6R_3$ and $R_2 \to R_2 + 3R_3$, we get

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 1 & 3/2 \\ -5/4 & -1/4 & -3/4 \\ -1/4 & -1/4 & -1/4 \end{bmatrix} A$$

This is of the form $I_3 = BA$ which gives

$$A^{-1} = \begin{bmatrix} 3 & 1 & 3/2 \\ -5/4 & -1/4 & -3/4 \\ -1/4 & -1/4 & -1/4 \end{bmatrix}$$

12) Solve the System of linear Equations by Matrix Method $x+y+z=6$, $2x+3y-2z=2$, $5x+y+2z=13$?

Sol: let $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & -2 \\ 5 & 1 & 2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ $B = \begin{bmatrix} 6 \\ 2 \\ 13 \end{bmatrix}$

$\Rightarrow$ The given equations can be written in Matrix form as $\boxed{AX=B}$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & -2 \\ 5 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 2 \\ 13 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -4 \\ 0 & -4 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ -10 \\ -17 \end{bmatrix} \quad (\text{Applying } R_2 - 2R_1,\ R_3 - 5R_1)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & -4 \\ 0 & 0 & -19 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 16 \\ -10 \\ -57 \end{bmatrix} \quad (\text{Applying } \cancel{R_1 - 2R_2},\ R_3 + 4R_2)\ (R_1 - R_2)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 16 \\ -10 \\ 3 \end{bmatrix} \quad \text{Applying } \frac{R_3}{-19}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad (\text{Applying } R_1 - 5R_3,\ R_2 + 4R_3)$$

Hence the Solution $\boxed{x=1,\ y=2,\ z=3}$

14) If $A = \begin{bmatrix} 3 & 7-4i & -2+5i \\ 7+4i & -2 & 3+i \\ -2-5i & 3-i & 4 \end{bmatrix}$ then Show that A is Hermetetion and iA is Skew-Hermitetion.

Sol: Given $A = \begin{bmatrix} 3 & 7-4i & -2+5i \\ 7+4i & -2 & 3+i \\ -2-5i & 3-i & 4 \end{bmatrix}$

$$\therefore \bar{A} = \begin{bmatrix} 3 & 7+4i & -2-5i \\ 7-4i & -2 & 3-i \\ -2+5i & 3+i & 4 \end{bmatrix}$$

Thus $(\bar{A})^T = \begin{bmatrix} 3 & 7-4i & -2+5i \\ 7+4i & -2 & 3+i \\ -2-5i & 3-i & 4 \end{bmatrix} = A$

$\therefore$ Hence A is Hermitetion Matrix.

Let $B = iA$,

$$A = \begin{bmatrix} 3i & 4+7i & -5-2i \\ -4+7i & -2i & -1+3i \\ 5-2i & 1+3i & 4i \end{bmatrix}$$

Now $\bar{B} = \begin{bmatrix} -3i & 4-7i & -5+2i \\ -4-7i & 2i & -1-3i \\ 5+2i & 1-3i & -4i \end{bmatrix}$

$\therefore (\bar{B})^T = \begin{bmatrix} -3i & -4-7i & 5+2i \\ 4-7i & 2i & 1-3i \\ -5+2i & -1-3i & -4i \end{bmatrix}$

$= (-1)\begin{bmatrix} 3i & 4+7i & -5-2i \\ -4+7i & -2i & -1+3i \\ 5-2i & 1+3i & 4i \end{bmatrix} = -B$

Thus B i.e. iA is a skew-Hermitian Matrix.

1) Discuss for what values of λ, μ the simultaneous equations $x+y+z=6$, $x+2y+3z=10$, $x+2y+\lambda z=\mu$ have

i) No Solution

ii) a unique Solution

iii) an infinite number of Solution.

Solution:- The Matrix form of given System of Equation is

$$AX = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & \lambda \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \\ \mu \end{bmatrix} = B$$

We have the augmented Matrix is $[A/B] = \begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & \lambda & \mu \end{bmatrix}$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$, we get $[A/B] \sim \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 1 & \lambda-1 & \mu-6 \end{bmatrix}$

Applying $R_3 \to R_3 - R_2$, we get $[A/B] \sim \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \lambda-3 & \mu-10 \end{bmatrix}$

given i) Case:-

· let $\lambda \neq 3$ then Rank of $A = 3$ and Rank of $[A/B] = 3$, So they have Same Rank. It is Consistent. Here the Number of unknown is 3 which is Same as the Rank of A. The System of equations will have a unique Solution. Thus is true for any Value of μ.

∴ Thus if $\lambda \neq 3$ and μ has Any Value, the given System of Equations will have a unique Solution.

ii) Case:- Suppose $\lambda = 3$, and $\mu \neq 10$, then we can See that Rank of $A = 2$ and Rank of $[A/B] = 3$, Since Rank of $[A]$ and $[A/B]$ are not equal, we say that the System of equations has no Solution (inconsistent).

iii) Case:- Let $\lambda = 3$ and $\mu = 10$, then we have Rank of A = Rank of $[A/B]$ $= 2$

∴ The given System of equations will be Consistent.

But here the Number of unknown $= 3 >$ rank of A.

Hence the system has infinitely Many Solutions.

15) find whether the following System of equations are Consistent. If so Solve them

[$x+2y+2z=2$, $3x-2y-z=5$, $2x-5y+3z=-4$, $x+4y+6z=0$].

Sol: The given equations can be written in the matrix form as $AX=B$

i.e
$$\begin{bmatrix} 1 & 2 & 2 \\ 3 & -2 & -1 \\ 2 & -5 & 3 \\ 1 & 4 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ -4 \\ 0 \end{bmatrix}$$

The Augmented Matrix $[A, B] = \begin{bmatrix} 1 & 2 & 2 & 2 \\ 3 & -2 & -1 & 5 \\ 2 & -5 & 3 & -4 \\ 1 & 4 & 6 & 0 \end{bmatrix}$

Applying $R_2 \to R_2 - 3R_1$, $R_3 \to R_3 - 2R_1$, $R_4 \to R_4 - R_1$ we get $[A, B] \sim$

$$\begin{bmatrix} 1 & 2 & 2 & 2 \\ 0 & -8 & -7 & -1 \\ 0 & -9 & -1 & -8 \\ 0 & 2 & 4 & -2 \end{bmatrix}$$

Apply $R_3 \to 8R_3 - 9R_2$ and $R_4 \to 4R_4 + R_2$, we get

$$[A, B] \sim \begin{bmatrix} 1 & 2 & 2 & 2 \\ 0 & -8 & -7 & -1 \\ 0 & 0 & 55 & -55 \\ 0 & 0 & 9 & -9 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 2 & 2 \\ 0 & -8 & -7 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$ (Applying $\left[\frac{R_3}{55}, \frac{R_4}{9}\right]$)

$$\sim \begin{bmatrix} 1 & 2 & 2 & 2 \\ 0 & -8 & -7 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$ (Applying $R_4 \to R_4 - R_3$)

Since RANK OF A = 3 and Rank of [A, B] = 3,

∴ given system is Consistent and it has solution.

∴ The given system has a unique solution.

We have
$$\begin{bmatrix} 1 & 2 & 2 \\ 0 & -8 & -7 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ -1 \\ 0 \end{bmatrix}$$

~~$x+2y+2z=2$~~ Now values of x, y, z.

$\Rightarrow x+2y+2z=2$ —— (1)

$\Rightarrow -8y-7z=-1$ —— (2)

$\Rightarrow \boxed{z=-1}$ —— (3)

(3) in (2)

$\cancel{-8y+7(-1)=-1}$

$\cancel{-8y-7=-1}$

$\cancel{-8y=-1+7}$

$\cancel{-8y=6}$

$\cancel{y=6/8=3/4}$

$\cancel{y=-3/4}$

$-8y-7(-1)=-1$

$-8y=-8$

$\boxed{y=1}$ —— (4)

Now (3), (4) in (1)

$x+2(1)+2(-1)=2$

$x+2-2=2$

$\boxed{x=2}$

$\therefore \boxed{x=2,\ y=1,\ z=-1}$

17) Find the value of λ for which the system of equation $3x-y+4z=3$, $x+2y-3z=-2$, $6x+5y+\lambda z=-3$ will have infinite number of solutions and solve them with that λ value.

Sol) The given system of equation can be written in the matrix form as

$AX=B$ i.e. $\begin{bmatrix}3 & -1 & 4\\1 & 2 & -3\\6 & 5 & \lambda\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}3\\-2\\-3\end{bmatrix}$.

The Augmented matrix is $[A,B]=\begin{bmatrix}3 & -1 & 4 & 3\\1 & 2 & -3 & -2\\6 & 5 & \lambda & -3\end{bmatrix}$

Applying $R_2\to 3R_2-R_1$, $R_3\to R_3-2R_1$, $[AB]\sim\begin{bmatrix}3 & -1 & 4 & 3\\0 & 7 & -13 & -9\\0 & 7 & \lambda-8 & -9\end{bmatrix}$

Applying $R_3\to R_3-R_2$, $[AB]\sim\begin{bmatrix}3 & -1 & 4 & 3\\0 & 7 & -13 & -9\\0 & 0 & \lambda+5 & 0\end{bmatrix}$

If $\lambda=-5$, Rank of $A=2$ and Rank of $[A,B]=2$

Number of unknowns $=3$

$\therefore$ Rank of $A=$ Rank of $[A,B]\neq$ number of unknowns

Hence when $\lambda=-5$, the given system is consistent and it has an infinite number of solution.

If $\lambda=-5$ the given system becomes $\begin{bmatrix}3 & -1 & 4\\0 & 7 & -13\\0 & 0 & 0\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}3\\-9\\0\end{bmatrix}$

$3x-y+4z=3$ — ① and $7y-13z=-9$ — ②

let $z=K$. Then from ② we get

$7y-13K=-9 \Rightarrow 7y=13K-9 \Rightarrow y=(13K-9)/7$

substituting the value of y in ①, we get

$3x-\frac{1}{7}(13K-9)+4K=3 \Rightarrow 3x=\frac{13}{7}K-4K+3-\frac{9}{7}$

$\Rightarrow 3x=-\frac{15}{7}K+\frac{12}{7} \Rightarrow x=\frac{1}{7}(-5K+4)$

$\therefore$ The solution is $x=\frac{1}{7}(-5K+4)$, $y=\frac{1}{7}(13K-9)$, $z=K$

13) Prove that the following set of equations are consistent and solve them. $3x+3y+2z=1$, $x+2y=4$, $10y+3z=-2$, $2x-3y-z=5$

sol) The given system of equations can be written in the matrix form as follows

$$AX = \begin{bmatrix} 3 & 3 & 2 \\ 1 & 2 & 0 \\ 0 & 10 & 3 \\ 2 & -3 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ -2 \\ 5 \end{bmatrix} = B$$

The Augumented matrix of the given equation is

$$[A/B] = \begin{bmatrix} 3 & 3 & 2 & 1 \\ 1 & 2 & 0 & 4 \\ 0 & 10 & 3 & -2 \\ 2 & -3 & -1 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 0 & 4 \\ 3 & 3 & 2 & 1 \\ 0 & 10 & 3 & -2 \\ 2 & -3 & -1 & 5 \end{bmatrix} \quad [\text{Applying } R_1 \leftrightarrow R_2]$$

$$\begin{bmatrix} 1 & 2 & 0 & 4 \\ 0 & -3 & 2 & -11 \\ 0 & 10 & 3 & -2 \\ 0 & -7 & -1 & -3 \end{bmatrix} \quad [\text{Applying } R_2 - 3R_1 \text{ and } R_4 - 2R_1]$$

$$\begin{bmatrix} 1 & 2 & 0 & 4 \\ 0 & 1 & -2/3 & 11/3 \\ 0 & 10 & 3 & -2 \\ 0 & -7 & -1 & -3 \end{bmatrix} \quad \left[\text{Applying } \frac{R_2}{-3}\right]$$

$$\begin{bmatrix} 1 & 2 & 0 & 4 \\ 0 & 1 & -2/3 & 11/3 \\ 0 & 0 & 29/3 & -116/3 \\ 0 & 0 & -17/3 & 68/3 \end{bmatrix} \quad [\text{Applying } R_3 - 10R_2 \text{ and } R_4 + 7R_2]$$

$$\begin{bmatrix} 1 & 2 & 0 & 4 \\ 0 & 1 & -2/3 & 11/3 \\ 0 & 0 & 1 & -4 \\ 0 & 0 & -17/3 & 68/3 \end{bmatrix} \quad \left[\text{Applying } \tfrac{3}{29} R_2\right]$$

$$\begin{bmatrix} 1 & 2 & 0 & 4 \\ 0 & 1 & -2/3 & 11/3 \\ 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \left[\text{Applying } R_4 + \tfrac{17}{3} R_3\right]$$

Thus the matrix $[A/B]$ has been reduced to Echelon form.

$\therefore$ Rank $[A/B]$ = no. of non-zero rows = 3

By the same row operations, we have

$$A \sim \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & -2/3 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore$ Rank $(A) = 3$

since Rank (A) = Rank $[A/B] = 3$, therefore the given equations are consistent.

Also rank $(A) = 3$ = no. of unknowns.

Hence the given equations have unique solution

The given equations are equivalent to the equations

$$x + 2y = 4, \quad y - \frac{2}{3} z = \frac{11}{3}, \quad z = -4$$

on solving these equation, we get

$$x = 2, \; y = 1, \; z = -4$$

18) Solve the system of equation

$x + 2y + 3z = 1, \; 2x + 3y + 8z = 2, \; x + y + z = 3$

sol) let $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 8 \\ 1 & 1 & 1 \end{bmatrix}, \; X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \; B = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

Then system can be written as $AX = B$

Consider $[A, B] = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 2 & 3 & 8 & 2 \\ 1 & 1 & 1 & 3 \end{bmatrix}$

$\begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & -1 & 2 & 0 \\ 0 & -1 & -2 & 2 \end{bmatrix}$ [Applying $R_2 - 2R_1$, $R_3 - R_1$]

$\begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & -1 & 2 & 0 \\ 0 & 0 & -4 & 2 \end{bmatrix}$ [Applying $R_3 - R_2$]

This is Echelon form. Number of non-zero rows is 3

$\therefore \rho[A, B] = 3$

Now $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & 2 \\ 0 & 0 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$

Number of non-zero rows is 3

$\therefore$ rank $(A) = \rho(A) = 3$

$\therefore \rho(A, B) = \rho(A) = 3$

$\therefore$ The above system has unique solution

Now solve $AX = B$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & 2 \\ 0 & 0 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$$

$\Rightarrow x + 2y + 3z = 1 \quad - (1)$

$-y + 2z = 0 \quad - (2)$

$-4z = 2 \quad - (3)$

By back substitution

$(3) \Rightarrow z = \frac{-1}{2}$

$(2) \Rightarrow -y + 2\left(\frac{-1}{2}\right) = 0 \Rightarrow -y - 1 = 0 \Rightarrow y = -1$

Now $x+2y+3z=1$ gives

$$x+2(-1)+3\left(\frac{-1}{2}\right)=1 \Rightarrow x-2-\frac{3}{2}=1 \Rightarrow x=\frac{9}{2}$$

Thus $X = \begin{bmatrix} \frac{9}{2} \\ -1 \\ -\frac{1}{2} \end{bmatrix}$ is the unique solution

29) Find the values of P and q so that the equation $2x+3y+5z=9$, $7x+3y+2z=8$, $2x+3y+Pz=q$ have

i) No solution   ii) unique solution   iii) An infinit no. of sol

sol) Given equation are $2x+3y+5z=9$, $7x+3y+2z=8$, $2x+3y+Pz=q$

The equation can be written in the matrix form $AX=B$

i.e. $$\begin{bmatrix} 2 & 3 & 5 \\ 7 & 3 & 2 \\ 2 & 3 & P \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 9 \\ 8 \\ q \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & 3 & 5 \\ 0 & -15 & -31 \\ 0 & 0 & P-5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 9 \\ -47 \\ q-9 \end{bmatrix}$$ [Applying $2R_2-7R_1$, R_3-R_1]

Applying same operation, we get

$$[A/B] \sim \begin{bmatrix} 2 & 3 & 5 & 9 \\ 0 & -15 & -31 & -47 \\ 0 & 0 & P-5 & q-9 \end{bmatrix}$$

we have $\det(A) = \begin{vmatrix} 2 & 3 & 5 \\ 7 & 3 & 2 \\ 2 & 3 & P \end{vmatrix}$

$$= 2(3P-6)-3(7P-4)+5(21-6)$$

$$= 6P-12-21P+12+75 = -15P+75$$

$\det A = 0 \Rightarrow P=5$

Case I :- when $p=5, q \neq 9$

The rank$(A)=2$ and rank$[A,B]=3$

The system ~~we~~ will be inconsistent. The system will not have any solution.

Case II :- when $p \neq 5$ det $A \neq 0$

The system will have unique solution

Case III :- when $p=5, q=9$

rank$(A)=2$ and rank$[A,B]=2$

since rank$(A)=$ rank$[A,B] <$ number of variables $=3$

$\therefore$ The system will be consistent and will have infinite number of solution.

21) Find weather the following system of equations are consistent. If so, solve them. (2011)

$x+2y-z=3,\ 3x-y+2z=-1,\ 2x-2y+3z=2,\ x-y+z=-1$

Sol: Given system of equation is $x+2y-z=3$, $3x-y+2z=-1$, $2x-2y+3z=2$, $x-y+z=-1$

The given system can be written in matrix form $AX=B$

ie

$$\Rightarrow \begin{bmatrix} 1 & 2 & -1 \\ 3 & -1 & 2 \\ 2 & -2 & 3 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \\ 2 \\ -1 \end{bmatrix}$$

$R_2 \to R_2 - 3R_1$ $\quad$ ~~$R_2 \to 3R_1$~~

$R_3 \to R_3 - 2R_1$

$R_4 \to R_4 - R_1$

$$\Rightarrow \begin{bmatrix} 7 & 0 & 3 \\ 0 & -7 & 5 \\ 0 & 0 & 5 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -10 \\ 32 \\ 2 \end{bmatrix}$$

$R_1 \to 7R_1 + 2R_2$, $7R_4 - 3R_2$

$R_3 \to 7R_3 - 6R_2$

$$\Rightarrow \begin{bmatrix} 7 & 0 & 3 \\ 0 & -7 & 5 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -10 \\ \frac{32}{5} \\ -2 \end{bmatrix}$$

$R_3 \to \frac{R_3}{5}$, $\frac{R_4}{-1}$

$$\Rightarrow \begin{bmatrix} 7 & 0 & 3 \\ 0 & -7 & 5 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -10 \\ 32/5 \\ -42/5 \end{bmatrix}$$

Number of non zero rows in $A = 3$. Thus rank$(A) = 3$

No. of non zero rows in $[A,B] = 4$. Thus rank $[A,B] = 4$

$\therefore$ Rank $(A) \neq$ Rank $[A,B]$

Hence the given system of equation is inconsistent. ie(no solutions).

2Q. Determine whether the following equations will have a solution. If so, solve them. (2011)

$x_1 + 2x_2 + x_3 = 2$ $\quad\quad$ $3x_1 + x_2 - 2x_3 = 1$

$4x_1 - 3x_2 - x_3 = 3$ $\quad\quad$ $2x_1 + 4x_2 + 2x_3 = 4$

Sol: writing the given equations in matrix form $AX = B$, we have

$$\Rightarrow \begin{bmatrix} 1 & 2 & 1 \\ 3 & 1 & -2 \\ 4 & -3 & -1 \\ 2 & 4 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \\ 4 \end{bmatrix}$$

$R_2 \to R_2 - 3R_1$

$R_3 \to R_3 - 4R_1$

$R_4 \to R_4 - 2R_1$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 1 \\ 0 & -5 & -5 \\ 0 & -11 & -5 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -5 \\ -5 \\ 0 \end{bmatrix}$$

$R_2 \to R_2/-5$

$$A \sim \begin{bmatrix} 1 & 1 & -2 & 3 \\ 0 & -3 & 3 & 4 \\ 0 & -3 & 3 & 4 \\ 0 & -3 & 3 & 4 \end{bmatrix} \quad \begin{matrix} R_3 \to R_3 - R_2 \\ R_4 \to R_4 - R_2 \end{matrix}$$

$$A \sim \begin{bmatrix} 1 & 1 & -2 & 3 \\ 0 & -3 & 3 & -4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

This is in the Echelon form. we have rank$(A) = 2$

Since rank$(A) = 2$ is less than the no. of unknowns $(=4)$, therefore, the given system has infinite number of non-trivial solutions.

$\therefore$ no of independent solutions $= 4-2 = 2$

Now, we shall asign arbitrary values to 2 variables and the remaining 2 variables shall be found in terms of these. The given system of equations is equivalent to

$$\begin{bmatrix} 1 & 1 & -2 & 3 \\ 0 & -3 & 3 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

This gives the equations $x+y-2z+3w=0$, $-3y+3z-4w=0$

Taking $z=\lambda$ and $w=\mu$, we see that $x=\lambda-\frac{5}{3}\mu$, $y=\lambda-\frac{4}{3}\mu$, $z=\lambda$, $w=\mu$ constitutes the general solution of the given system.

25. Test for consistency and if consistent solve the system,

$5x+3y+7t=4$, $3x+26y+2t=9$, $7x+2y+10t=5$

Sol: $A = \begin{bmatrix} 5 & 3 & 7 \\ 3 & 26 & 2 \\ 7 & 2 & 10 \end{bmatrix}$ $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ $B = \begin{bmatrix} 4 \\ 9 \\ 5 \end{bmatrix}$

The argument matrix is $[A|B] = \begin{bmatrix} 5 & 3 & 7 & 4 \\ 3 & 26 & 2 & 9 \\ 7 & 2 & 10 & 5 \end{bmatrix}$ $5R_2 - 3R_1$, $5R_3 - 7R_1$

$$\sim \begin{bmatrix} 5 & 3 & 7 & 4 \\ 0 & 121 & -11 & 33 \\ 0 & -11 & 1 & -3 \end{bmatrix} \quad R_2 \to R_2/11$$

$$\sim \begin{bmatrix} 5 & 3 & 7 & 4 \\ 0 & 11 & -1 & 3 \\ 0 & -11 & 1 & -3 \end{bmatrix} \quad R_3 \to R_3 + R_2$$

$$\sim \begin{bmatrix} 5 & 3 & 7 & 4 \\ 0 & 11 & -1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Number of non-zero rows = 2

Rank $(A) = 2 =$ Rank $(A|B)$

$\therefore$ given system is consistent. No. of variables $= 3$ $\therefore$ no of solutions is infinite

From matrix

$5x + 3y + 7z = 4$

$11y - z = 3$

Let $z = k$. Then $11y = 3 + k \Rightarrow y = \frac{3+k}{11}$

$5x = 4 - 3y - 7z = 4 - 3\left(\frac{3+k}{11}\right) - 7k$

$= \frac{44 - 9 - 3k - 77k}{11} = \frac{53 - 80k}{11}$

$\Rightarrow x = \frac{53 - 80k}{55}$

$\therefore$ This will give infinite number of solutions.

$$\Rightarrow \begin{bmatrix} 3 & 4 & 5 \\ 4 & 5 & 6 \\ 0 & -2 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a \\ b \\ 3c-5a \end{bmatrix}$$

$R_2 \to 3R_2 - 4R_1$

$$\Rightarrow \begin{bmatrix} 3 & 4 & 5 \\ 0 & -1 & -2 \\ 0 & -2 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a \\ 3b-4a \\ 3c-5a \end{bmatrix}$$

$R_3 \to R_3 - 2R_2$

$$\Rightarrow \begin{bmatrix} 3 & 4 & 5 \\ 0 & -1 & -2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a \\ 3b-4a \\ 3a-6b+3c \end{bmatrix}$$

$\therefore$ From the matrix we can have $3a+3c=6b \Rightarrow \underline{\underline{a+c=2b}}$

Q3: Solve completely the system of equations

$x+y-2z+3w=0 \qquad x-2y+z-w=0$

$4x+y-5z+8w=0 \qquad 5x-7y+2z-w=0$

Sol: The given system of equations in matrix form is

$$AX = \begin{bmatrix} 1 & 1 & -2 & 3 \\ 1 & -2 & 1 & -1 \\ 4 & 1 & -5 & 8 \\ 5 & -7 & 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = 0$$

$R_2 \to R_2 - R_1$, $R_4 \to R_4 - 5R_1$

$R_3 \to R_3 - 4R_1$

$$A \approx \begin{bmatrix} 1 & 1 & -2 & 3 \\ 0 & -3 & 3 & 4 \\ 0 & -3 & 3 & 4 \\ 0 & -12 & 12 & -16 \end{bmatrix}$$

$R_4 \to R_4/4$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ 0 & -11 & -5 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ -5 \\ 6 \end{bmatrix}$$

$R_1 \to R_1 - 2R_2$

$R_3 \to R_3 + 11R_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 6 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 6 \\ 0 \end{bmatrix}$$

we can observe that A is in Echelon form. No. of non zero rows = 3

Rank $(A) = 3 =$ Rank $[A, B]$ = no. of variables

The system is consistent and solution is unique.

$6x_3 = 6 \Rightarrow x_3 = 1$

$x_2 + x_3 = 1 \Rightarrow x_2 = 0$

$x_1 - x_3 = 0 \Rightarrow x_1 = x_3 = 1$

$\therefore$ The solution is $x_1 = 1, x_2 = 0, x_3 = 1$

29. Show that the equation $3x + 4y + 5z = a$, $4x + 5y + 6z = b$ and $5x + 6y + 7z = c$ do not have a solution unless at $a + c = 2b$ (2015)

Sol: writing the given system in matrix form $AX = B$ we get

$$\Rightarrow \begin{bmatrix} 3 & 4 & 5 \\ 4 & 5 & 6 \\ 5 & 6 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$R_3 \to 3R_3 - 5R_1$

(27) (8) Solve the system of equations

$x+y+w=0, \quad y+z=0, \quad x+y+z+w=0, \quad x+y+2z=0$

The eqn can be written in matrix form as $\boxed{AX=0}$

where $A = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 0 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}, \quad 0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

consider $A = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 0 \end{bmatrix}$ $R_3 \to R_3 - R_1$

$R_4 \to R_4 - R_1$

$\begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 2 & -1 \end{bmatrix}$ $R_4 \to R_4 - 2R_3$

$\begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$

Rank (A) = 4 and Number of variables = 4

Therefore, there is no non-zero soln.

Hence $x = y = z = 0$ is the only solution.

26. Examine whether the vectors are linearly dependent or not $(3,1,1), (2,0,-1), (4,2,1)$

Let $a(3,1,1) + b(2,0,-1) + c(4,2,0) = 0$

$\Rightarrow 3a + 2b + 4c = 0$ —— (1)

$\Rightarrow a + 2c = 0$ —— (2)

$\Rightarrow a - b + c = 0$ —— (3)

cosider $\begin{bmatrix} 3 & 2 & 4 \\ 1 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix}$

$R_3 \to R_3$ (Applying $R_3 - R_1$)

$\begin{bmatrix} 3 & 2 & 4 \\ 1 & 0 & 2 \\ 0 & -1 & -1 \end{bmatrix}$

This is in Echelon form. No of non-zero rows is 3

$\therefore$ Rank = 3

No. of variables = 3

$\therefore$ No. of non-zero solution = 3 - 3 = 0

$a = b = c = 0$ is the only solution

$\therefore$ The three vectors are linearly independent.

(29) Determine the values of λ for which the following set of eqn may posses non-trivial soln

$$3x_1 + x_2 - \lambda x_3 = 0$$
$$4x_1 - 2x_2 - 3x_3 = 0$$
$$2\lambda x_1 + 4x_2 + \lambda x_3 = 0$$

For each permissible value of λ, determine the general solution.

The given system of eqn is equivalent to the matrix eqn $\boxed{AX = 0}$

$$AX = \begin{bmatrix} 3 & 1 & -\lambda \\ 4 & -2 & -3 \\ 2\lambda & 4 & \lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

The given system posses non-trival soln, if rank of $A <$ number of unknowns. i.e., Rank of $A < 3$

$$\begin{vmatrix} 3 & 1 & -\lambda \\ 4 & -2 & -3 \\ 2\lambda & 4 & \lambda \end{vmatrix} = 0$$

$$\Rightarrow 3(-2\lambda + 12) - 1(4\lambda + 6\lambda) - \lambda(16 + 4\lambda) = 0$$

$$\Rightarrow -4\lambda^2 - 32\lambda + 36 = 0$$

$$\Rightarrow \lambda^2 + 8\lambda - 9 = 0$$

$$\Rightarrow (\lambda + 9)(\lambda - 1) = 0$$

$$\therefore \lambda = -9 \quad (\text{or}) \quad \lambda = 1$$

Case-1: For $\boxed{\lambda=-9}$, the given system reduces to

$$3x_1+x_2+9x_3=0$$
$$4x_1-2x_2-3x_3=0$$
$$-18x_1+4x_2-9x_3=0$$

Now rank of $A=2<3$ (no. of variables) $\left[\because \begin{vmatrix} 3 & 1 \\ 4 & -2 \end{vmatrix}=-10\neq 0\right]$

$\therefore$ System has infinite no. of solns.

$\therefore$ Number of independent solns $=3-2=1$

Let $x_1=2K$ and from the first two eqns, we get

$$x_2+9x_3=-6K \text{ and } -2x_2-3x_3=-8K$$

on solving $x_2=6K$ and $x_3=-\frac{4}{3}K$, we get

$\boxed{x_1=2K}$, $\boxed{x_2=6K}$ and $\boxed{x_3=-4/3\,K}$ as the general soln of the given system

Case-2: For $\boxed{\lambda=1}$, the given system reduces to

$$3x_1+x_2-x_3=0$$
$$4x_1-2x_2-3x_3=0$$
$$2x_1+4x_2+x_3=0$$

Now rank of $A=2<3$ (number of variables)

Hence the system has infinite number of solns.

$\therefore$ No. of independent soln $=3-2=1$

Let $x_1=K$ and from the first two eqns we get

$$x_2-x_3=3K \text{ and } -2x_2-3x_3=-4K$$

on solving, $x_2=-K$ and $x_3=2K$ where K is a constant

$\therefore$ $\boxed{x_1=K}$, $\boxed{x_2=-K}$ and $\boxed{x_3=2K}$ is the general soln of the given system

(58) Show that the only real number λ for which the system

$x + 2y + 3z = \lambda x$; $3x + y + 2z = \lambda y$; $2x + 3y + z = \lambda z$

has non-zero soln is 6 and solve them, when $\lambda = 6$

Given system can be expressed as $AX = O$

$$A = \begin{bmatrix} 1-\lambda & 2 & 3 \\ 3 & 1-\lambda & 2 \\ 2 & 3 & 1-\lambda \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } O = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Here number of variables $= n = 3$

The given system of eqn possess a non-zero (non-trivial) soln, if

Rank of $A <$ number of unknown i.e., Rank of $A < 3$

For this we must have $\det A = 0$

$$\therefore \begin{vmatrix} 1-\lambda & 2 & 3 \\ 3 & 1-\lambda & 2 \\ 2 & 3 & 1-\lambda \end{vmatrix} = 0$$

Applying $R_1 \to R_1 + R_2 + R_3$

$$\begin{vmatrix} 6-\lambda & 6-\lambda & 6-\lambda \\ 3 & 1-\lambda & 2 \\ 2 & 3 & 1-\lambda \end{vmatrix} = 0$$

$$(6-\lambda)\begin{vmatrix} 1 & 1 & 1 \\ 3 & 1-\lambda & 2 \\ 2 & 3 & 1-\lambda \end{vmatrix} = 0$$

Applying $C_2 \to C_2 - C_1$

$C_3 \to C_3 - C_1$

$$(6-\lambda)\begin{vmatrix} 1 & 0 & 0 \\ 3 & -2-\lambda & -1 \\ 2 & 1 & -1-\lambda \end{vmatrix} = 0$$

$$(6-\lambda)\left[(-2-\lambda)(-1-\lambda)+1\right] = 0$$

$\Rightarrow (6-\lambda)(\lambda^2+3\lambda+3)=0 \Rightarrow \boxed{\lambda=6}$ is the only real value and other values are complex when $\lambda=6$, the given system becomes

$$\begin{bmatrix} -5 & 2 & 3 \\ 3 & -5 & 2 \\ 2 & 3 & -5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_2 \to 5R_2 + 3R_1$

$R_3 \to 5R_3 + 2R_1$

$$\begin{bmatrix} -5 & 2 & 3 \\ 0 & -19 & 19 \\ 0 & 19 & -19 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_3 \to R_3 + R_2$

$$\begin{bmatrix} -5 & 2 & 3 \\ 0 & -19 & 19 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow -5x + 2y + 3z = 0$ and $-19y + 19z = 0 \Rightarrow \boxed{y = z}$

Since Rank of $A <$ number of unknows

(Number of unknows $= 3$, Rank of $A = 2$)

$\therefore$ The given system has infinite number of non-trivial solns.

Let $\underline{z = k} \Rightarrow \underline{y = k}$ and $-5x + 2k + 3k = 0$

$$-5x + 5k = 0$$

$$\cancel{5}x = \cancel{5}k$$

$$\underline{x = k}$$

$\therefore x = k, y = k, z = k$ is the solution.

(31) find the value of 'α' show that the vectors (1, 1, 0) (1, α, 0) and (1,1,1) are linearly dependent. (2018)

Sol

~~Let a(1,1,1) + b (1, α.~~

Let $(1,1,0) = a(1,\alpha,0) + b(1,1,1)$

Comparing the components

$a + b = 1$ — ①

$a\alpha + b = 1$ — ②

$b = 0$ — ③

sub $b=0$ in eqn ①

$a + 0 = 1$

$a = 1$.

sub $a=1$, $b=0$ in eqn ②

$a(\alpha) + b = 1$

$1(\alpha) + 0 = 1$

$\alpha + 0 = 1$

$\boxed{\alpha = 1}$

(32) Determine wheather the vectors (1, 2, 3), (2, 3, 4), (3, 4, 5) are linearly dependent or not. (2017)

Sol

let $(1,2,3) = a(2,3,4) + b(3,4,5)$

comparing the coefficients

$2a + 3b = 1$ — ①

$3a + 4b = 2$ — ②

$4a + 5b = 3$ — ③

By solving ① & ③ we get

$$2(2a+3b=1) \Rightarrow 4a+6b=2$$
$$4a+5b=3$$
$$\boxed{b=-1}$$

sub $b=-1$ in eqⁿ ②

$$3a+4b=2$$
$$3a+4(-1)=2$$
$$3a-4=2$$
$$3a=2+4$$
$$3a=6$$
$$\boxed{a=2}$$

Thus $a=2, b=-1$ is ~~sto~~ satisfying the third eqⁿ

∴ The three vectors are dependent.

(33) Express the following system in matrix form and solve by gauss-elimination method

$2x_1+x_2+2x_3+x_4=6$; $6x_1-6x_2+6x_3+12x_4=36$;

$4x_1+3x_2+3x_3-3x_4=-1$; $2x_1+2x_2-x_3+x_4=10$.

Sol: The Augumented Matrix

$$\begin{bmatrix} 2 & 1 & 2 & 1 & 6 \\ 6 & -6 & 6 & 12 & 36 \\ 4 & 3 & 3 & -3 & -1 \\ 2 & 2 & -1 & 1 & 10 \end{bmatrix}$$

$$R_2 \to \frac{R_2}{6} \qquad \begin{bmatrix} 2 & 1 & 2 & 1 & 6 \\ 1 & -1 & 1 & 2 & 6 \\ 4 & 3 & 3 & -3 & -1 \\ 2 & 2 & -1 & 1 & 10 \end{bmatrix}$$

$R_1 \leftrightarrow R_2$

$$\begin{bmatrix} 1 & -1 & 1 & 2 & 6 \\ 2 & 1 & 2 & 1 & 1 \\ 4 & 3 & 3 & -3 & -1 \\ 2 & 2 & -1 & 1 & 10 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1, \; R_3 \to R_3 - 4R_1$

$R_4 \to R_4 - 2R_1$

$$\begin{bmatrix} 1 & -1 & 1 & 2 & 6 \\ 0 & 3 & 0 & -3 & -6 \\ 0 & 7 & -1 & -4 & -25 \\ 0 & 4 & -3 & -33 & -2 \end{bmatrix}$$

$R_3 \to 3R_3 - 7R_2$

$R_4 \to 3R_4 - 4R_2$

$$\begin{bmatrix} 1 & -1 & 1 & 2 & 6 \\ 0 & 3 & 0 & -3 & -6 \\ 0 & 0 & -3 & -12 & -33 \\ 0 & 0 & -9 & 3 & 18 \end{bmatrix}$$

$R_4 \to R_4 - 3R_3$

$$\begin{bmatrix} 1 & -1 & 1 & 2 & 6 \\ 0 & 3 & 0 & -3 & -6 \\ 0 & 0 & -3 & -12 & -33 \\ 0 & 0 & 0 & 39 & 117 \end{bmatrix}$$

$x_1 - x_2 + x_3 + 2x_4 = 6$ — (1)

$3x_2 - 3x_4 = -6$ — (2)

$-3x_3 - 12x_4 = -33$ (3)

$39x_4 = 117$ — (4)

$x_4 = 3.$

Sub x_4 in (3) & (2) we get

$x_3 = -1$ & $x_2 = 1$

substitute the values of x_2, x_3 & x_4 in (1) we get

$x_1 - 1 - 1 + 6 = 6 \Rightarrow x_1 = 2$.

$\therefore$ The soln is $x_1 = 2,\ x_2 = 1,\ x_3 = -1,\ x_4 = 3$.

34. Q9 use gauss- elimination method to solve $x+2y-3z=9$, $2x-y+z=0$

$4x-y+z=4$.

Sol:- Asugmented matrix.

$$A=\begin{bmatrix} 1 & 2 & -3 & 9 \\ 2 & -1 & z & 0 \\ 4 & -1 & z & 4 \end{bmatrix}$$

$=$ $R_2 \to R_2-2R_1$

$R_3 \to R_3-4R_1$

$$\begin{bmatrix} 1 & 2 & -3 & 9 \\ 0 & 6 & -8 & 18 \\ 0 & 7 & -13 & 32 \end{bmatrix}$$

$R_3 \to 6R_3 - 7R_2$.

$$\begin{bmatrix} 1 & 2 & -3 & 9 \\ 0 & 6 & -8 & 18 \\ 0 & 0 & 2 & 6 \end{bmatrix}$$

$x+2y-3z=9$

$2y-\cancel{8z}\quad 6y-8z=18$

$2z=6$

$\boxed{z=3}$

$6y-8(3)=18$

$6y-24=18$

$6y=18+24$

$6y=42$

$\boxed{y=7}$

$x+2(7)-3(3)=9$

$x+14-9=9$

$x=9+9-14$

$\boxed{x=4}$

30) solve using gauss- seidal Iteration method.

ii) $x_1+10x_2+x_3=6$

$10x_1+x_2+x_3=6$ (1)

$x_1+x_2+10x_3=6$

Sol :-

$x_1+10x_1+x_3=6$ (2)

$10x_1+x_2+x_3=6$ (1)

$x_1+x_2+10x_3=6$ (3)

Ist Iteration $x_1=\frac{1}{10}(6-x_2-x_3)$

put $x_2=0,\ x_3=0$

$\frac{1}{10}(6-0-0)$

$=\frac{6}{10}=0.6$

$x_2=\frac{1}{10}(6-x_1-x_3)$

$=\frac{1}{10}(6-0.6-0)$

$=\frac{5.4}{10}=0.54$

$x_3=\frac{1}{10}(6-x_1-x_2)$

$=\frac{1}{10}(6-0.6-0.5)$

$=0.49$

IInd Iteration $x_1=\frac{1}{10}[6-x_2-x_3]$

$\frac{1}{10}[6.05-0.4]$

$$= \frac{5.1}{10} = 0.51$$

$$x_2 = \frac{1}{10}[6-0.5-0.4]$$

$$= \frac{5.1}{10} = 0.51$$

$$x_3 = \frac{1}{10}[6-0.5-0.5]$$

$$= \frac{5}{10} = 0.5$$

III^rd Iteration

$$x_1 = \frac{1}{10}[6-0.5-0.5]$$

$$= \frac{5}{10} = 0.5$$

$$x_2 = \frac{1}{10}[6-0.5-0.5]$$

$$= \frac{5}{10} = 0.5$$

$$x_3 = \frac{1}{10}[6-0.5-0.5]$$

$$= \frac{5}{10}$$

$$= 0.5$$

IV Iteration

$$x_1 = \frac{1}{10}[6-0.5-0.5]$$

$$= \frac{5}{10} = 0.5$$

$$x_2 = \frac{1}{10}[6-0.5-0.5]$$

$$= \frac{5}{10} = 0.5$$

$$x_3 = \frac{1}{10}[6-0.5-0.5]$$

$$= \frac{5}{10} = 0.5.$$

S.NO	variable	I^{st} Iteration	II^{nd} Iteration	III^{rd} Iteration	IV Iteration
1.	x_1	0.6	0.51	0.5	0.5
2.	x_2	0.54	0.51	0.5	0.5
3.	x_3	0.49	0.5	0.5	0.5

3) solve the following system of equations by using gauss-seidal method correct to three decimal place.

$$8x-3y+2z=20$$
$$4x+11y-z=33$$
$$6x+3y+12z=35$$

sol:

$$8x-3y+2z=20$$
$$4x+11y-z=33$$
$$6x+3y+12z=35$$

I^{st} Iteration

$$x=\frac{1}{8}[20+3y-2z]$$

put $y=0, z=0$

$$\frac{1}{8}(20+3(0)-2(0)]=\frac{20}{8}=2.5$$

$$y=\frac{1}{11}(33-4x+z]$$

$$\frac{1}{11}[33-4(2.5)+0]=\frac{23}{11}=2.09$$

$$z=\frac{1}{12}[35-6x-3y]$$

$$\frac{1}{12}[35-6(2.5)-3(2.09)=\frac{13.73}{12}=1.14.$$

$\underset{\sim}{\text{II}}^{nd}$ Iteration

$$x=\frac{1}{8}\left[20+3y-2z\right]=\frac{1}{8}\left[20+3(2.09)-2(1.14)\right]=\frac{23.99}{8}$$
$$=2.99$$

$$y=\frac{1}{11}\left[33-4x+z\right]=\frac{1}{11}\left[33-4(2.99)+1.14\right]=\frac{22.18}{11}=2.01$$

$$z=\frac{1}{12}\left[35-6x-3y\right]=\frac{1}{12}\left[35-6(2.99)-3(2.01)\right]=\frac{11.03}{12}=0.91$$

$\underset{\sim}{\text{III}}^{rd}$ Iteration

$$x=\frac{1}{8}\left[20+3y-2z\right]=\frac{1}{8}\left[20+3(2.01)-2(0.91)\right]=\frac{24.21}{8}=3.02$$

$$y=\frac{1}{11}\left[33-4(3.02)+0.91\right]=\frac{21.83}{11}=1.98$$

$$z=\frac{1}{12}\left[35-6x-3y\right]=\frac{1}{12}\left[35-6(3.02)-3(1.98)\right]=\frac{10.94}{12}=0.91$$

$\underset{\sim}{\text{IV}}^{th}$ Iteration

$$x=\frac{1}{8}\left[20+3y-2z\right]=\frac{1}{8}\left[20+3(1.98)-2(0.91)\right]=\frac{24.12}{8}=3.01$$

$$y=\frac{1}{11}\left[33-4x+z\right]=\frac{1}{11}\left[33-4(3.01)+0.91\right]=\frac{21.87}{11}=1.98$$

$$z=\frac{1}{12}\left[35-6(3.01)-3(1.98)\right]=\frac{11}{12}=0.91$$

$\underset{\sim}{\text{V}}^{th}$ Iteration

$$x=\frac{1}{8}\left[20+3y-2z\right]=\frac{1}{8}\left[20+3(1.98)-2(0.91)\right]=\frac{24.12}{8}=3.01$$

$$y=\frac{1}{11}\left[33-4x+z\right]=\frac{1}{11}\left[33-4(3.01)+0.91\right]=\frac{21.87}{11}=1.98$$

$$z=\frac{1}{12}\left(35-6x-3y\right]=\frac{1}{12}\left[35-6(3.01)-3(1.98)\right]=\frac{11}{12}=0.91$$

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